

RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021

FIRST YEAR (BATCH 2020-23)

STATISTICS (GENERAL)

Date : 14/08/2021

Time : 11.00 am – 2.00 pm

Paper : II

Full Marks : 75

Group - A

Answer **any six** questions of the following :

[6×5]

1. If $X_1, X_2, \dots, X_n$ is a random sample from $N(\mu, \sigma^2)$, how are the following distributed?

i) $\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{\sigma^2}$

ii) $\frac{\bar{X} - \mu}{S/\sqrt{n}}$, where $S^2 = \frac{1}{n-1} \sum (X_i - \bar{X})^2$ (2+3)

2. If $X_1, X_2, \dots, X_n$ is a random sample from $N(\theta, 1)$ then show that $\left(\bar{X}^2 - \frac{1}{n}\right)$ is an unbiased estimator of θ^2 .

3. If T is a consistent estimator of θ , show that $\frac{nT}{n+1}$ is also a consistent estimator of θ . [n is the number of sample observations]

4. In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of the perfectness of the coin is accepted if and only if at most 3 heads are obtained. What is the probability of type-I error?

5. Explain why the maximum observations $X_{(n)}$ [amongst the observations $X_1, X_2, \dots, X_n$ drawn from the uniform distribution $u(0, \theta)$] is the MLE of θ ? The p.d.f of the uniform variable is as follows :

$$f(x) = \begin{cases} \frac{1}{\theta} & ; 0 < x \leq \theta \\ 0 & ; \text{otherwise} \end{cases}$$

6. Suppose there is a population which is normally distributed with mean μ and variance σ^2 , both unknown. How would you set up an interval for the unknown population variance σ^2 using sample statistics?
7. Explain how Fisher's t statistic is arrived at.

Group - B

Answer **any four** questions of the following :

[4×5]

8. What purpose is served by an Index number? How is the base period of an Index numbers chosen? Explain what do you understand by 'Analysis of Variance'. State the basic assumptions in an ANOVA.

9. If Bowley's and Fisher's price indices are denoted by x and y respectively, then show that Laspeyres' index (L) and Paasche's index (P) are given by

Either

$$L = x + \sqrt{x^2 - y^2} \text{ and } P = x - \sqrt{x^2 - y^2}$$

Or

$$L = x - \sqrt{x^2 - y^2} \text{ and } P = x + \sqrt{x^2 - y^2}$$

10. Explain why Fisher's price index number formula is called an ideal index. In connection with price index numbers, explain the terms 'homogeneity error' and 'formula error'.
11. If, in every stratum, the simple estimator $\bar{y}_h$ is unbiased, then show that $\bar{y}_{st} = \sum_{h=1}^L W_h \bar{y}_h$ is unbiased for population mean $\bar{y}$, where W_h is the proportion of population units in the strata and L denotes the total number of strata in the population.
Derive the sampling variance of $\bar{y}_{st}$ and state how you would unbiasedly estimate the same.
12. Write the advantages of Stratified Random Sampling over Simple Random Sampling.
What do you understand by the Seasonal Variations in a time series? Give Example.

Group - C

Answer **any one** question from the following :

[1×10]

13. Two types of batteries are tested for their lengths of life and the following data are obtained.

	Sample size	Mean life in hours	Sample variance
Type - A	9	600	121
Type - B	8	640	144

Is there a significant difference in the two means? [Value of t for 15 df. at 5% level is 2.131]

14. A sample analysis of certain examination results of 200 students in a school was made. It was found that 46 students failed. 68 secured third division, 62 secured second divisions, and the rest were placed in the first division. Are there results commensurate with the annual examination results which are in the ratio 2:3:3:2 for the above categories respectively? (The tabulated value of chi-square for 3 df. at 5% level of significance is 7.81)

Group - D

Answer **any one** question from the following :

[1×5]

15. Given that the cost of living index number for 2004 with 1991 as base year is 250, will Mr. X be satisfied if his income rises to 12000 in 2004 as compared to Rs. 5000 in 1991.
16. Consider a population of $N = 6$ units with values 1, 2, 3, 4, 5 and 6.
- Write down all possible samples of size 2 drawn by SRSWOR scheme. Verify that the sample mean is unbiased for the population mean.

ii) Also compute the sampling variance of the sample mean. (3+2)

Answer **any one** question from the following : [1×10]

17. Below are given the yields in gm. Per plot (plot size = $\frac{1}{2000}$ acre) for three variables of seed cotton:

Variety 1	Variety 2	Variety 3
77	109	46
70	106	70
63	137	71
84	079	65
95	134	61
81	078	40
88	126	47
101	098	73

- Write out the analysis of variance table.
- Test if the varieties differ significantly among themselves.
- If the result of (ii) is affirmative, determine which varieties differ.

18. Given below are the figures on production (in thousand metric tons) of a cooperative sugar factory:

Year	2010	2011	2012	2013	2014	2015	2016
Production	77	88	84	85	91	98	90

Fit a linear trend by least squares method. Tabulate the trend values. Compute the monthly estimated increase in production during the period.

TABLE VI
RANDOM SAMPLING NUMBERS *

4652	3819	8431	2150	2352	2472	0043	3488
9031	7617	1220	4129	7148	1943	4890	1749
2030	2327	7353	6007	9410	9179	2722	8445
0641	1489	0828	0385	8488	0422	7209	4950
8479	6062	5593	6322	9439	4996	1322	4918
9917	3490	5533	2577	4348	0971	2580	1943
6376	9899	9259	5117	1336	0146	0680	4052
7287	0983	3236	3252	0277	8001	6058	4501
0592	4912	3457	8773	5146	2519	3931	6794
6499	9118	3711	8838	0691	1425	7768	9544
0769	1109	7909	4528	8772	1876	2113	4781
8678	4873	2061	1835	0954	5026	2967	6560
0178	7794	6488	7364	4094	1649	2284	7753
3392	0963	6364	5762	0322	2592	3452	9002
0264	6009	1311	5873	5926	8597	9051	8995
4089	7732	8163	2798	1984	1292	0041	2500
9376	7365	7987	1937	2251	3411	6737	0367
3039	3780	2137	7641	4030	1604	2517	9211
8971	8653	1855	5285	5631	2649	6696	5475
0373	4153	5199	5765	2067	6627	3100	5716
9092	4773	0002	7000	7800	2292	2933	6125
2464	1038	3163	3569	7155	2029	2538	7080
3027	6215	3125	5856	9543	3660	0255	5544
5754	9247	1164	3283	1865	5274	5471	1346
4358	3716	6949	8502	1573	5763	5046	7135
7178	8324	8379	7365	4577	4864	0629	5100
5035	5939	3665	2160	6700	7249	1738	2721
3318	0220	3611	9887	4608	8664	2185	7290
9058	1735	7435	6822	6622	8286	8901	5534
7886	5182	7595	0305	4903	3306	8088	3899
3354	8454	7386	1333	5345	6565	3159	3991
3415	7671	0846	7100	1790	9449	6285	2525
3918	5872	7898	6125	2268	1898	0755	6034
6138	9045	6950	8843	6533	0917	6673	5721
3825	1704	2835	4677	4637	7329	3156	3291
1349	0417	9311	9787	1284	0769	8422	1077
4234	0248	7760	6504	2754	4044	0842	9080
6880	3201	7044	3657	5263	0374	7563	6599
0714	5008	5076	1134	5342	1608	5179	0967
3448	6421	3304	0583	1260	0662	7257	0766
5711	7343	7539	3684	9397	5335	4031	1486
2588	3301	0553	2427	3598	2580	7017	9176
8581	4253	7404	5264	5411	3431	3092	8573
8475	6322	3949	9675	6533	1133	8776	2216
0272	5624	8549	5552	7469	2799	2822	9620
7383	7795	7939	2652	4456	6993	2950	8573
5126	2089	7729	0945	3901	4445	7117	8186
2064	3760	0939	7319	5939	3432	2030	4752
9315	8185	7805	6294	7072	6491	4012	1016
6814	8752	3462	6001	3302	3895	7371	3432