RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, AUGUST 2021

FIRST YEAR (BATCH 2020-23)

Date : 14/08/2021 Time : 11.00 am - 2.00 pm STATISTICS (GENERAL) Paper : II

Full Marks : 75

[6×5]

Group - A

Answer **any six** questions of the following :

- 1. If $X_1, X_2, ..., X_n$ is a random sample from $N(\mu, \sigma^2)$, how are the following distributed?
 - i) $\sum_{i=1}^{n} \left(X_{i} \overline{X} \right)^{2} \sigma^{2}$

ii)
$$\frac{\overline{X} - \mu}{S/\sqrt{n}}$$
, where $S^2 = \frac{1}{n-1} \sum \left(X_i - \overline{X} \right)^2$ (2+3)

- 2. If $X_1, X_2, ..., X_n$ is a random sample from $N(\theta, 1)$ then show that $\left(\overline{X}^2 \frac{1}{n}\right)$ is an unbiased estimator of θ^2 .
- 3. If T is a consistent estimator of θ , show that $\frac{nT}{n+1}$ is also a consistent estimator of θ . [*n* is the number of sample observations]
- 4. In order to test whether a coin is perfect, the coin is tossed 5 times. The null hypothesis of the perfectness of the coin is accepted if and only if at most 3 heads are obtained. What is the probability of type-I error?
- 5. Explain why the maximum observations $X_{(n)}$ [amongst the observations $X_1, X_2, ..., X_n$ drawn from the uniform distribution $u(0, \theta)$] is the MLE of θ ? The p.d.f of the uniform variable is as follows :

$$f(\mathbf{x}) = \frac{1}{\theta} ; \quad 0 < x \le \theta$$
$$= 0 ; \text{ otherwise}$$

- 6. Suppose there is a population which is normally distributed with mean μ and variance σ^2 , both unknown. How would you set up an interval for the unknown population variance σ^2 using sample statistics?
- 7. Explain how Fisher's *t* statistic is arrived at.

Group - B

Answer **any four** questions of the following :

8. What purpose is served by an Index number? How is the base period of an Index numbers chosen? Explain what do you understand by 'Analysis of Variance'. State the basic assumptions in an ANOVA.

[4×5]

9. If Bowley's and Fisher's price indices are denoted by x and y respectively, then show that Laspeyres' index (L) and Paasche's index (P) are given by

Either

$$L = x + \sqrt{x^2 - y^2}$$
 and $P = x - \sqrt{x^2 - y^2}$
Or

$$L = x - \sqrt{x^2 - y^2}$$
 and $P = x + \sqrt{x^2 - y^2}$

- 10. Explain why Fisher's price index number formula is called an ideal index. In connection with price index numbers, explain the terms 'homogeneity error' and 'formula error'.
- 11. If, in every stratum, the simple estimator \overline{y}_h is unbiased, then show that $\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y}_h$ is unbiased for population mean \overline{y} , where W_h is the proportion of population units in the strata and L denotes the total number of strata in the population.

Derive the sampling variance of \overline{y}_{st} and state how you would unbiasedly estimate the same.

12. Write the advantages of Stratified Random Sampling over Simple Random Sampling.What do you understand by the Seasonal Variations in a time series? Gives Example.

Group - C

Answer **any one** question from the following :

13. Two types of batteries are tested for their lengths of life and the following data are obtained.

	Sample size	Mean life in hours	Sample variance		
Type - A	9	600	121		
Type - B	8	640	144		

Is there a significant difference in the two means? [Value of t for 15 df. at 5% level is 2.131]

14. A sample analysis of certain examination results of 200 students in a school was made. It was found that 46 students failed. 68 secured third division, 62 secured second divisions, and the rest were placed in the first division. Are there results commensurate with the annual examination results which are in the ratio 2:3:3:2 for the above categories respectively? (The tabulated value of chi-square for 3 df. at 5% level of significance is 7.81)

Group - D

Answer **<u>any one</u>** question from the following :

- 15. Given that the cost of living index number for 2004 with 1991 as base year is 250, will Mr. X be satisfied if his income rises to 12000 in 2004 as compared to Rs. 5000 in 1991.
- 16. Consider a population of N = 6 units with values 1, 2, 3, 4, 5 and 6.
 - i) Write down all possible samples of size 2 drawn by SRSWOR scheme. Verify that the sample mean is unbiased for the population mean.

[1×10]

[1×5]

ii) Also compute the sampling variance of the sample mean.

Answer **any one** question from the following :

17. Below are given the yields in gm. Per plot (plot size = $\frac{1}{2000}$ acre) for three variables of seed cotton:

Variety 1	Variety 2	Variety 3
77	109	46
70	106	70
63	137	71
84	079	65
95	134	61
81	078	40
88	126	47
101	098	73

i) Write out the analysis of variance table.

ii) Test if the varieties differ significantly among themselves.

iii) If the result of (ii) is affirmative, determine which varieties differ.

18. Given below are the figures on production (in thousand metric tons) of a cooperative sugar factory:

Year	2010	2011	2012	2013	2014	2015	2016
Production	77	88	84	85	91	98	90

Fit a linear trend by least squares method. Tabulate the trend values. Compute the monthly estimated increase in production during the period.

		RANDON	TABL SAMPI	E VI	ADED.C.		
4652 9031 2030 0641 8479	3819 7617 2327 1489 6062	8431 1220 7353 0828 5593	2150 4129 6007 0385 6322	2352 7148 9410 8488 9439	2472 1943 9179 0422 4996	0043 4890 2722 7209 1322	3488 1749 8445 4950
9917 6376 7287 0592 6499	3490 9899 0983 4912 9118	5533 9259 3236 3457 3711	2577 5117 3252 8773 8838	4348 1336 0277 5146 0691	0971 0146 8001 2519 1425	2580 0680 6058 3931 7768	1943 4052 4501 6794 9544
0769 8678 0178 3392 0264	1109 4873 7794 0963 6009	7909 2061 6488 6364 1311	4528 1835 7364 5762 5873	8772 0954 4094 0322 5926	1876 5026 1649 2592 8597	2113 2967 2284 3452 9051	4781 6560 7753 9002 8995
4089 9376 3039 8971 0373	7732 7365 3780 8653 4153	8163 7987 2137 1855 5199	2798 1937 7641 5285 5765	1984 2251 4030 5631 2067	1292 3411 1604 2649 6627	0041 6737 2517 6696 3100	2500 0367 9211 5475 5716
9092 2464 3027 5754 4358	4773 1038 6215 9247 3716	0002 3163 3125 1164 6949	7000 3569 5856 3283 8502	7800 7155 9543 1865 1573	2292 2029 3660 5274 5763	2933 2538 0255 5471 5046	6125 7080 5544 1346 7135
7178 5035 3318 9058 7886	8324 5939 0220 1735 5182	8379 3665 3611 7435 7595	7365 2160 9887 6822 0305	4577 6700 4608 6622 4903	4864 7249 8664 8286 3306	0629 1738 2185 8901 8088	5100 2721 7290 5534 3899
3354 3415 3918 6138 3825	8454 7671 5872 9045	7386 0846 7898 6950 2835	1333 7100 6125 8843 4677	5345 1790 2268 6533 4637	6565 9449 1898 0917 7329	3159 6285 0755 6673 3156	3991 2525 6034 5721 3291
1349 4234 6880 0714	0417 0248 3201 5008	9311 7760 7044 5076	9787 6504 3657 1134 0583	1284 2754 5263 5342 1260	0769 4044 0374 1608 0662	8422 0842 7563 5179 7257	1077 9080 6599 0967 0766
5711 2588 8581 8475	7343 3301 4253 6322	7539 0553 7404 3949	3684 2427 5264 9675	9397 3598 5411 6533	5335 2580 3431 1133 2799	4031 7017 3092 8776 2822	1486 9176 8573 2216 9620
7383 5126 2064 9315	5624 7795 2089 3760 8185	8549 7939 7729 0939 7805	5552 2652 0945 7319 6294	4456 3901 5939 7072	6993 4445 3432 6491 3895	2950 7117 2030 4012 7371	8573 8186 4752 1016 3432
	4652 9031 2030 0641 8479 9917 6376 7287 0592 6499 0769 8678 3039 8971 0373 9092 2464 3027 5754 4358 7178 5035 3318 9092 2464 3027 5754 4358 7178 5035 3318 9058 7886 3354 3354 3415 3918 6138 3825 1349 4234 6880 0714 3448 5711 2588 8581 8475 0272 7383 5126 2064 9315	4652 3819 9031 7617 2030 2327 0641 1489 8479 6062 9917 3490 6376 9899 7287 0983 0592 4912 6499 9118 0769 1109 8678 4873 0178 7794 3392 0963 0264 6009 4089 7732 9376 7365 3039 3780 8971 8653 3027 6215 5754 9247 4358 3716 7178 8324 5035 5939 3318 0220 9058 1735 7886 5182 3354 8454 3415 7671 3918 5872 6138 9045 3825 1704 1349 0417 4234 0248 6880	RANDON 4652 3819 8431 9031 7617 1220 2030 2327 7353 0641 1489 0828 8479 6062 5593 9917 3490 5533 6376 9899 9259 7287 0983 3236 0592 4912 3457 6499 9118 3711 0769 1109 7909 8678 4873 2061 0178 7794 6488 3392 0963 6364 0264 6009 1311 4089 7732 8163 9376 7365 7987 3039 3780 2137 8971 8653 1855 0373 4153 5199 9092 4773 0002 2464 1038 3163 3027 6215 3125 5754 924	4652 3819 8431 2150 9031 7617 1220 4129 2030 2727 1220 4129 9031 7617 1220 4129 2030 2727 1220 4129 9051 2427 1489 9828 6087 8479 6062 5593 6322 9917 3490 5233 2577 6376 9899 9259 5117 7287 0983 3236 3252 0599 9118 3711 8838 0769 1109 7909 4528 8678 4873 2061 1835 3392 0963 6364 5762 0264 6009 1311 5873 4089 7732 8163 2798 9376 7365 7987 1937 3039 3780 2137 7641 8971 86533 1855 52856 <td>TABLE VI RANDOM SAMPLING NUM 9031 23227 7353 6007 2410 9030 23227 7353 6007 9410 9041 1489 9828 0385 8488 96376 3490 5533 2577 4348 6376 9899 9239 6322 9439 9376 3990 5533 2577 4348 6376 4873 2061 1835 0954 0178 7794 6488 7365 0954 0178 7794 6488 7365 0954 0178 7794 6488 7365 0954 0392 4937 2061 1835 0954 0392 0963 6364 5762 0322 0264 6009 1311 5873 5926 4089 7732 8163 2798 1984 3039 3765 7987 1937 2251 9076<td>TABLE VI RANDOM SAMPLING NUMBERS 46552 3819 8431 2150 2352 2472 2030 2327 7353 6007 9410 9179 3041 1489 08228 0385 84488 0422 8479 6062 5593 6322 9439 4996 9917 3490 5533 2577 4348 0971 7287 0989 92239 2577 4348 0971 6499 9118 3711 8538 0691 1425 0769 9109 7909 4528 8772 1876 86778 4873 2061 1835 0954 5026 0373 7365 7987 1937 5226 8597 4089 7732 8163 2798 1984 1292 9376 7365 7987 1937 5251 3631 90373 4153 5199 5765 2067 6627</td><td>TABLE YI RANDOM SAMPLING NUMBERS - 4657 2030 2030 2041 2041 2041 2041 2041 2041 2041 204</td></td>	TABLE VI RANDOM SAMPLING NUM 9031 23227 7353 6007 2410 9030 23227 7353 6007 9410 9041 1489 9828 0385 8488 96376 3490 5533 2577 4348 6376 9899 9239 6322 9439 9376 3990 5533 2577 4348 6376 4873 2061 1835 0954 0178 7794 6488 7365 0954 0178 7794 6488 7365 0954 0178 7794 6488 7365 0954 0392 4937 2061 1835 0954 0392 0963 6364 5762 0322 0264 6009 1311 5873 5926 4089 7732 8163 2798 1984 3039 3765 7987 1937 2251 9076 <td>TABLE VI RANDOM SAMPLING NUMBERS 46552 3819 8431 2150 2352 2472 2030 2327 7353 6007 9410 9179 3041 1489 08228 0385 84488 0422 8479 6062 5593 6322 9439 4996 9917 3490 5533 2577 4348 0971 7287 0989 92239 2577 4348 0971 6499 9118 3711 8538 0691 1425 0769 9109 7909 4528 8772 1876 86778 4873 2061 1835 0954 5026 0373 7365 7987 1937 5226 8597 4089 7732 8163 2798 1984 1292 9376 7365 7987 1937 5251 3631 90373 4153 5199 5765 2067 6627</td> <td>TABLE YI RANDOM SAMPLING NUMBERS - 4657 2030 2030 2041 2041 2041 2041 2041 2041 2041 204</td>	TABLE VI RANDOM SAMPLING NUMBERS 46552 3819 8431 2150 2352 2472 2030 2327 7353 6007 9410 9179 3041 1489 08228 0385 84488 0422 8479 6062 5593 6322 9439 4996 9917 3490 5533 2577 4348 0971 7287 0989 92239 2577 4348 0971 6499 9118 3711 8538 0691 1425 0769 9109 7909 4528 8772 1876 86778 4873 2061 1835 0954 5026 0373 7365 7987 1937 5226 8597 4089 7732 8163 2798 1984 1292 9376 7365 7987 1937 5251 3631 90373 4153 5199 5765 2067 6627	TABLE YI RANDOM SAMPLING NUMBERS - 4657 2030 2030 2041 2041 2041 2041 2041 2041 2041 204

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